INTERNATIONAL CONFERENCE ON TOPOLOGICAL ALGEBRAS AND THEIR APPLICATIONS 2018

JANUARY 25–28

2018

TALLINN UNIVERSITY

Tallinn 2018

International Conference on

Topological Algebras and their Applications

ICTAA 2018

January 25–28
2018

Tallinn University
Tallinn
Estonia

SCHEDULE OF "ICTAA 2018"

Thursday, January 25th

9.30–10.00 Registration at the lobby of the Astra building

10.00–10.30 Opening ceremony at room **A-543**
10.30–11.00 Coffee Break at the cafeteria "Oaas" at the Terra building

Talks will take place at room **T-409**

11.00–11.50 **Wiesław Żelazko**, What is known and what is not known about maximal abelian subalgebras of Banach algebras

12.00–14.00 Lunch break

14.00–14.20 **Malazela Salthiel Maepa**, On subsymmetric renormings of galbed algebras
14.25–14.55 **Mati Abel**, About a result of G. A. Allan
15.00–15.20 **Paul Tammo**, Coincidence of topological Jacobson radicals

15.30–16.00 Coffee break near **T-409**

The evening session will take place at room **A-543**

16.00–17.00 Problem session
17.00–17.30 **Lourdes Palacios** (via Skype from Mexico), A characterization of C*-normed algebras via positive functionals

Friday, January 26th

Talks of the whole day will take place at **T-409**

9.00–9.50 **Hugo Arizmendi Peimbert**, On the subalgebra A_0 of the bounded elements of $C(t)$
10.00–10.20 **Maria Stella Adamo**, Derivations and representable functionals over Banach quasi *-algebras

10.30–11.00 Coffee break near **T-409**

11.00–11.30 **Tomasz Ciaś**, Fréchet algebras with a dominating Hilbert algebra norm
11.40–12.10 **Reyna María Pérez Tiscareño**, On locally pseudoconvex Q-algebras

12.10–14.00 Lunch break

14.00–14.20 **Camillo Trapani**, Recent results on locally convex quasi *-algebras
14.25–15.15 **Maria Fragoulopoulou**, Some interactions among topological algebras, geometry and physics

15.30 Possibility for a guided tour in the old city of Tallinn

Saturday, January 27th

Talks of the whole morning will take place at **T-409**

9.00–9.50 **Alexander Ya. Helemskii**, Multi-normed spaces, based on non-discrete measures, and their tensor products

10.00–10.20 **Peng Cao**, Scattered elements in Banach Jordan and associative algebras

10.30–11.00 Coffee break near **T-409**

11.00–11.30 **Martin Weigt**, On nuclear generalized B^*-algebras

11.40–12.10 **Mart Abel**, On a category of Topological Segal algebras

12.20–14.00 Lunch break

Afternoon session will take place at **A-543**

14.00–14.50 **Wiesław Żelazko**, A short history of Polish mathematics

15.00–15.20 Closing ceremony

18.00–21.00 Conference dinner at the cafeteria "Oaas" in Terra building

Sunday, January 28th

9.45–10.00 Gathering at the lobby of Mare building for the excursion

10.00–18.00 Excursion

ABSTRACTS OF THE TALKS OF "ICTAA 2018"

On a category of Topological Segal algebras

Mart Abel
Tallinn University/University of Tartu

The topic of Segal algebras in the context of C^*-algebras, Banach algebras or Fréchet algebras has been popular for several decades. In 2017, we offered a new approach to Segal algebras, which could be used for all topological algebras.

Definition. A topological algebra (A, τ_A) is a left (right or two-sided) Segal algebra in a topological algebra (B, τ_B) via an algebra homomorphism $f : A \to B$, if

1) $\mathrm{cl}_B(f(A)) = B$;
2) $\tau_A \supseteq \{f^{-1}(U) : U \in \tau_B\}$;
3) $f(A)$ is a left (respectively, right or two-sided) ideal of B.

In the present talk, we start with some results about the general topological Segal algebras. The main part of the talk will be focused on different possibilities of defining a category of general topological Segal algebras. We will also talk about some recent results about one specific category of topological Segal algebras and offer some ideas which could lead to some results for more general categories of topological Segal algebras.

About a result of G. A. Allan

Mati Abel
University of Tartu

In 1965 G. R. Allan stated the following: if A is a unital locally convex algebra with separately continuous multiplication and with continuous inversion, then

$$\mathrm{sp}(a) \subseteq \mathrm{sp}^r(a) \subseteq \mathrm{cl}(\mathrm{sp}(a)) \tag{1}$$

for each $a \in A$. If, in addition, A is pseudo-complete, then

$$\mathrm{sp}^r(a) = \mathrm{cl}(\mathrm{sp}(a)). \tag{2}$$

Here, $\mathrm{sp}(a)$ denotes the algebraic spectrum of a, $\mathrm{sp}^r(a)$ the regular spectrum of a (or Allan-Waelbroeck spectrum of a) and $\mathrm{cl}(U)$ denotes the closure of U in the topology of $\mathbb{C} \cup \{\infty\}$.

It is shown that (1) and (2) hold when the multiplication in A is jointly continuous. Generalizations of this result to nonunital and non locally convex cases are considered.

Derivations and representable functionals over Banach quasi *-algebras

Maria Stella Adamo
Università degli Studi di Catania

A Banach quasi *-algebra is a mathematical structure that may be seen as completion of a normed *-algebra with a coarser norm satisfying some conditions. An important tool to investigate their structure is given by representable functionals, i.e., those linear functionals that allow a GNS-like construction.

Derivations have been studied since last century because of the many links and applications on different fields of Mathematics and Physics.

The aim of the talk is to discuss about some results for derivations on Banach quasi *-algebras properly defined with the help of representable functionals and sesquilinear forms associated to them. It turns out that this kind of framework allows us to obtain well known results in a more general framework.

This is a joint work with Camillo Trapani.

On the subalgebra A_0 of the bounded elements of $C(t)$ endowed with different topologies

Hugo Arizmendi Peimbert
UNAM

Let $C(t)$ be algebra of all complex rational functions, J. H. Williamson gave to it a locally convex metrizable topology τ_W in which the addition and multiplication are continuous. We prove that the set A_0 of all bounded elements of $(C(t), \tau_W)$, in the sense of G. R. Allan, is a maximal subalgebra of $C(t)$ and a principal ideals Q-algebra. Also A_0 is the increasing countable union of Q-normed algebras.

We provide $C(t)$ with the weak topology $w^\sharp$ determined by its algebraic dual $C(t)^*$ and with the maximal locally convex topology $\tau_{LC}^{\max}$. In the two cases the multiplication is just separately continuous. We prove also $A_0 = \mathbb{C}$, with any of those topologies. Finally we describe the Allan and the extended spectra for any element of $C(t)$.

This is a joint work with Angel Carrillo and Pavel Ramos.

Scattered elements in Banach Jordan and associative algebras

Peng Cao
Beijing Institute of Technology

A Jordan or associative algebra is called scattered if it consists of elements with countable spectrum (so called scattered elements). It is proved that for Banach, Jordan or associative, algebras there exists the largest scattered ideal and it is closed. Accordingly, this determines the scattered topological radical. The characterization of the scattered radical is given. We also give the similar spectral characterizations of the kh-socle in Banach, Jordan or associative algebras.

Fréchet algebras with a dominating Hilbert algebra norm

Tomasz Ciaś
Adam Mickiewicz University in Poznań

A Fréchet space E with a fundamental sequence $(||\cdot||_n)_{n\in\mathbb{N}}$ of seminorms has the property (DN) if there is a continuous norm $||\cdot||$ on E such that for all $k\in\mathbb{N}$ there is $n\in\mathbb{N}$ and a constant $C>0$ such that

$$||x||_k^2 \le C||x||\,||x||_n$$

for all $x\in E$; each norm $||\cdot||$ with this property is called a *dominating norm*. The property (DN) plays a key role in the structure theory of nuclear Fréchet spaces – in 1977 Dietmar Vogt proved that a nuclear Fréchet space is isomorphic to a closed subspace of the space s of rapidly decreasing sequences if and only if it admits a dominating norm.

In this talk we consider Fréchet *-algebras E with unit which admit a dominating Hilbert algebra norm, i.e., a dominating Hilbert norm $||\cdot|| = \sqrt{(\cdot,\cdot)}$ such that

(i) $(xy,z) = (y,x^*z)$ for all $x,y,z\in E$;

(ii) $(y^*,x^*) = (x,y)$ for all $x,y\in E$;

(iii) for all $x\in E$ there is $C>0$ such that $||xy||\le C||y||$ for all $y\in E$.

Many classical commutative Fréchet *-algebras admit a dominating Hilbert algebra norm, e.g., the algebra $C^\infty[-1,1]$, the algebras $C^\infty(M)$ of smooth functions on smooth compact manifolds and the algebra $H(\mathbb{C})$ of entire functions. A Fréchet *-algebra with unit and a dominating norm satisfying conditions (i) and (iii) is called a βDN-algebra. These algebras appear in the structure of the noncommutative topological *-algebra $\mathscr{L}^*(s)$, i.e., the maximal $\mathscr{O}^*$-algebra of unbounded operators on ℓ_2 whose domain is the space s.

We showed that if E is a unital commutative Fréchet *-algebra isomorphic as a Fréchet space to a complemented subspace of s, then E is isomorphic, as a Fréchet *-algebra, to a closed *-subalgebra F of $\mathscr{L}^*(s)$ such that $F\subset\mathscr{L}(\ell_2)$ if and only if E is a βDN-algebra. This result may be seen as a step towards an analogue – in the context of algebras of smooth functions – of the celebrated Gelfand-Naimark theorem describing commutative C^*-algebras.

Some interactions among topological algebras, geometry and physics

Maria Fragoulopoulou
University of Athens

It was in 1962, when Borchers and Uhlmann used representations of topological *–algebras for the reformulation of the Wightman's axioms of quantum field theory. Only after their work the theory of unbounded operators has started developing. Another contribution of topological algebras is the one of the solution of a purely geometric problem, related with abstract differential geometry (for short adg), introduced by A. Mallios (1990). What Mallios noticed in his theory of adg is that smooth functions, are, in fact, not necessary in developing differential geometry. What is also essential, in this aspect, is that this theory is applicable to quantum physics. For instance, "particles" can be treated as "geometrical objects" without reference to any space in the usual sense; you simply have to apply methods of adg. In this talk, we shall discuss some results showing the interaction of topological algebras with physics and geometry.

Multi-normed spaces, based on non-discrete measures, and their tensor products

Alexander Ya. Helemskii
Moscow State University

It was A. Lambert who discovered a new type of structures, situated, in a sense, between normed spaces and (abstract) operator spaces. His definition was based on the notion of amplification of a normed space by means of spaces l_2^n. Afterwards several mathematicians investigated more general structure, "p-multi-normed space", introduced with the help of spaces $l_p^n; 1 \leqslant p \leqslant \infty$. In the present talk we pass from l_p to $L_p(X,\mu)$ with an arbitrary measure. This happened to be possible in the frame-work of the non-coordinate ("index-free") approach to the notion of amplification, equivalent in the case of a discrete counting measure to the approach in mentioned articles.

Two categories arise. One consists of amplifications by means of an arbitrary normed space, and another one consists of p-convex amplifications by means of $L_p(X,\mu)$. Each of them has its own tensor product of its objects whose existence is proved by a respective explicit construction. As a final result, we show that the p-convex tensor product has especially transparent form for the so-called minimal L_p-amplifications of L_q-spaces, where q is the conjugate of p. Namely, tensoring $L_q(Y,\nu)$ and $L_q(Z,\lambda)$, we get $L_q(Y \times Z, \nu \times \lambda)$.

On subsymmetric renormings of galbed algebras

Malezela Salthiel Maepa
University of Pretoria

Let $\mathbb{K}$ denote either $\mathbb{R}$ or $\mathbb{C}$, the fields of real or complex numbers respectively, $\mathbb{N} = \{0, 1, 2, \ldots\}$, $\mathbb{Z}^+ = \{1, 2, \ldots\}$, $s_{\mathbb{K}}$ the set of all sequences (α_n) with $\alpha_n \in \mathbb{K}$ for each $n \in \mathbb{N}$, $k > 0$, ℓ^k the set of all $(\alpha_n) \in s_{\mathbb{K}}$, for which the series

$$\sum_{\nu=0}^{\infty} |\alpha_\nu|^k$$

is convergent, ℓ^0 the set of all $(\alpha_n) \in s_{\mathbb{K}}$ such that $\{k \in \mathbb{N} : \alpha_k \neq 0\}$ is finite, $\ell = \ell^1 \setminus \ell^0$ and

$$\ell^{(0,1]} = \bigcap_{k \in (0,1]} \ell^k.$$

Let A be a topological algebra with a nontrivial topology τ , that is, $\tau \neq \{\emptyset, A\}$. We'll denote by $G(A)$ the set of all $(\alpha_n) \in s_{\mathbb{K}}$ with the following property: *for every neighbourhood O of zero in A there is another neighbourhood U of zero in A such that*

$$\left\{ \sum_{k=0}^{n} \alpha_k a_k : a_0, \ldots, a_n \in U \right\} \subset O$$

for each $n \in \mathbb{N}$. The topological algebra A is said to be a *galbed algebra* if there exists a sequence $(\alpha_n) \in \ell \cap G(A)$.

We study this property for a topological algebra in which the underlying space is a subsymmetrically normed space.

A characterization of C*-normed algebras via positive functionals

Lourdes Palacios
Universidad Autónoma Metropolitana - Iztapalapa

A functional f on an involutive algebra E is *positive* if $f(xx^*) \geq 0$ for all $x \in E$. It is known that C^*-algebras always have a large supply of positive functionals. There is even the following result:

Let $(E, \|.\|)$ be a unital C^*-algebra. Then, for every $z \in E$, there is a positive functional f such that $f(e) = 1$ and $f(zz^*) = \|z\|^2$.

In this talk we note that in fact this is a property that characterizes C^*-algebras in the frame of involutive Banach algebras; moreover, the same situation is examined in some normed and non normed topological algebras. This is done through the existence of enough specific positive functionals.

Joint work with:
Marina Haralampidou (University of Athens, Greece)
Mohamed Oudadess (Ecole Normale Superiéure, Rabat, Morocco)
Carlos Signoret Poillon (Universidad Autónoma Metropolitana - Iztapalapa, Mexico)

On locally pseudoconvex Q-algebras

Reyna María Pérez Tiscareño
University of Tartu

Some equivalent conditions for a locally m-pseudoconvex algebra to be a Q-algebra have been studied, the proofs given used the submultiplicative property of the elements of the family of pseudoseminorms (non-homogeneous seminorms) that define the topology for the algebra. In this talk will be given some equivalent conditions where the submultiplicativity is not necessary.

Coincidence of topological Jacobson radicals

Paul Tammo
University of Tartu

The left (right) topological radical of a topologiacal algebra A is the intersection $\text{rad}_l(A)$ ($\text{rad}_r(A)$) of kernels of all continuous irreducible representations (antirepresentations) of A on all Hausdorff linear spaces. It is an open question, posed by B. Yood in 1964, when $\text{rad}_l(A) = \text{rad}_r(A)$. We describe some classes of topological algebras for which these radicals coincide.

This is a joint work with Mart Abel and Mati Abel.

Recent results on locally convex quasi *-algebras

Camillo Trapani
Università di Palermo

A locally convex quasi *-algebra $(\mathfrak{A}[\tau], \mathfrak{A}_0)$ arise in natural way when $\mathfrak{A}$ is the completion of a given *-algebra $\mathfrak{A}_0$ with respect to a locally convex topology τ, if the left- and right multiplications are separately but not jointly continuous. We will focus our attention to the case when $\mathfrak{A}_0$ is a C^*-algebra and τ defines a topology coarser than that defined by the C^*-norm. This object, when certain coupling properties of the two topologies are satisfied, is called a *locally convex quasi C^*-algebra* and many results on its structure have been obtained by F. Bagarello, M. Fragoulopoulou, A. Inoue, S. Triolo and the author.

We will summarize some of them, considering in particular the problem of when some type of Gel'fand-Naimark representation theorems can be obtained. A recent result in this direction shows that a strongly *-semisimple (i.e., with *sufficiently many* tracial forms) locally convex quasi C^*-algebra $(\mathfrak{A}, \mathfrak{A}_0)$ can be realized as a space of measurable operators in the sense of Segal and Nelson.

The problem of determining under which conditions a locally convex quasi *-algebra $(\mathfrak{A}, \mathfrak{A}_0)$ can be made into a locally convex quasi C^*-algebra will also be shortly discussed. A key role in this respect is played by *bounded elements*; that is those which are mapped into bounded operators by every *-representation of $(\mathfrak{A}, \mathfrak{A}_0)$.

On nuclear generalized B^*-algebras

Martin Weigt
Nelson Mandela University

Generalized B^*-algebras (*GB*-algebras* for short) are locally convex topological *-algebras which are generalizations of C^*-algebras, and were first studied by G. R. Allan in [1]. Later, P. G. Dixon, in [2], refined the notion of GB^*-algebra to include non-locally convex algebras. A GB^*-algebra A is also an abstract unbounded operator algebra, and contains a C^*-algebra $A[B_0]$ dense in A. Besides C^*-algebras, examples of GB^*-algebras include inverse limits of C^*-algebras, called *pro-C*-algebras*, and non-commutative Arens algebras. GB^*-algebras in Dixon's sense also include various algebras of unbounded operators affiliated to a von Neumann algebra (which includes the algebra of all equivalence classes of complex-valued measurable functions on [0,1] equipped with the topology of convergence in measure).

Nuclear C^*-algebras and tensor products of C^*-algebras are well understood, and tensor products of GB^*-algebras were investigated for the first time in [3]. A nuclear GB^*-algebra is defined in [3] to be a GB^*-algebra A for which the C^*-algebra $A[B_0]$ is a nuclear C^*-algebra, and some characterizations and examples of such algebras can also be found in [3]. The well known Choi-Effros theorem states that a C^*-algebra A is nuclear if and only if there is a net of unital completely positive mappings $\phi_\lambda : A \to A$ such that $\phi_\lambda(x) \to x$ for all $x \in A$.

In this talk, a general investigation of completely positive mappings between GB^*-algebras is initiated, whereby the main emphasis is a generalization of the Choi-Effros theorem to GB^*-algebras. Some examples will also be given.

References

[1] G. R. Allan, *On a class of locally convex algebras*, Proc. London Math. Soc., **17**(1967), 91–114.

[2] P. G. Dixon, *Generalized B* -algebras*, Proc. London Math. Soc., **21**(1970), 693–715.

[3] M. Fragoulopoulou, A. Inoue and M. Weigt, *Tensor products of generalized B*-algebras*, J. Math. Anal. Appl., **420**(2014), 1787-1802.

What is known and what is not known about maximal abelian subalgebras (MAS) of Banach algebras

Wiesław Żelazko
Polish Academy of Sciences

Let $\mathscr{A}$ be a commutative Banach algebra. It is known (Gelfand) that it is an MAS in the algebra $B(A)$ of all bounded operators on a Banach space $\mathscr{A}$. But it can be a MAS in many other Banach algebras. In my talk I shall discuss two questions;

(1) In how many Banach algebras $\mathscr{A}$ can be a MAS?

(2) How many MAS contains a given (non-commutative) Banach algebra A?

My talk will consist of results, problems, conjectures and examples.

A short history of Polish mathematics

Wiesław Żelazko
Polish Academy of Sciences

In the talk I shall explain how Poland, having before 1900 no mathematical traditions, achieved in years 1920-39 quite reasonable position in some modern by then fields of mathematics. In particular I shall describe Lwow School (Banach, Steinhaus and others), Warsaw School (Sierpinski, Mazurkiewicz, Kuratowski, Borsuk, Tarski and others), also Wilno School (Zygmund, Marcinkiewicz), Poznan School (Orlicz) and Krakow School (Zaremba, Żorawski, Wazewski). If time permits I shall mention also the postwar period after 1945.